1. **(20 Pts):** For the data given in ***HW4\_Problem1.txt*** ( two comma separated values), find :
   1. Coefficients for the best-fit quadratic model and the RMS error.

***Solution:***

y = a0 + a1(x) + a2(x2)

Table

Description automatically generated

*Plots:*

* 1. Find the coefficients a0 and a1 if ***ymodel = a0\*x +a1\*sinh(x)*** and the RMS error

***Solution:***

*ymodel = a0\*x +a1\*sinh(x)*

*Graphical user interface, application, table, Excel

Description automatically generated*

*Plots:*

**PART C**

1. **(20 Pts):** For the data given in **HW4\_problem2.txt** (two comma separated values: X,Y), find:
   1. Best-fit cubic model for the entire regime (0 ≤ X ≤ 1)

***Solution:***

**Table

Description automatically generated**

**Plots:**

* 1. Two piece-wise cubic models, first fit, f1(x), is valid from 0 ≤ X ≤ 0.5 and the second fit, f2(x) is valid from: 0.5 < X ≤ 1. (Note: the point X=0.5 belongs to the first fit).
  2. For the fits determined in 3(b), Plot the two functions f1(X) and f2(X) and comment on the continuity (C0 and C1: data and slope continuity) of the two models at X = 0.5. If f2(X) is extrapolated to X = 0.5.

***Solution:***

First function from 0 – 0.5

Table

Description automatically generated

Second function from 0.5 – 1 (not including 0.5)

Table, Excel

Description automatically generated

**Plotting both the functions:**

When x = 0.5;

Chart

Description automatically generated with medium confidence

From the above plot and numerical computation, we can see that the functions do not coincide at x = 0.5. Also, they don’t have the same slope at x = 0.5. Hence, they cannot be termed as continuous.

1. **(30 Pts): For the data given in HW4\_Problem3.txt (two comma separated values: X, Y), model with two piece-wise polynomial models, a cubic function (f1(X)) to fit from 0 ≤ X ≤ 0.5 and a quartic (4th order - f2(X)) to fit from 0.5 ≤ X ≤ 1, such that f2(X) maintains both C0 and C1 continuity at X = 0.5.**

***Solution:***

Function 1: Cubic function coefficients

Graphical user interface, application, table, Excel

Description automatically generated

Plots:

Second Function: Quartic function coefficients

Graphical user interface, application, table, Excel

Description automatically generated

Table

Description automatically generated

In this, the values for df1(x) and df2(x) at x = 0.5 is almost similar and hence the slope is said to be same for both the functions. Also, the values for both the function at x=0.5 are almost same numerically and hence, the two functions can be said to be continuous which can be even verified by looking at the plot provided above.

1. **(30 Pts):** Determine polynomial interpolation, T(x) = Σ Ti Ni(x); where x = {0,..,1} and i=1...5; such that T(x=0) = T1, T(x=1) = T2, T(x=0.5) = T3, T(X=0.25)= T4 and T(x=0.75) = T5. Plot the five interpolation functions, Ni(x).

Given T1 = 100, T2 = 100, T3=160, T4=120, T5=130; Plot the temperature field, T(x) and the five interpolation functions (Ni(x), i=1…….5) Interpolation functions.